1. Introduction

Switching electric arc of high current density \(10^5 – 10^{10}\) \(A/m^2\) is relatively well known and widely described in the literature, whereas close to the arc low temperature discharges that fill up large space of the processing chamber are difficult for general mathematical description. In order to create the mathematical model of such a quasi-arc discharge, the following parameters shall be taken into account: laws of mass, momentum and energy conservation, Maxwell’s equation, Ohm's law and the equation of gas state.

In electric arc discharges two approaches are applied: channel and black box models. Among black box models, the Mayr and Cassie models are most known and turned out to be the most efficient in the analysis of the arc discharge.

The paper presents the modification [1] of the Mayr’s model enabling the analysis of the integrated power system [2], [3] of plasma reactor with gliding arc (Glidarc) applied in air pollution control processes [4].

2. Black box electric arc models

On the basis of energy balance in the dynamic state of arc for unit length of the arc the following equation can be derived:

\[
dW = Ei dt - P_0 dt = (P - P_0) dt
\]  
(1)

where: \(dW\) - arc energy gain, \(E\) - gradient of arc voltage, \(i\) - arc current, \(P_0\) - energy supplied to the arc from the power source, \(P dt\) - energy taken from the arc by the way of convection, conductivity and radiation.

Elementary arc conductance is expressed as follows

\[
G = \frac{1}{R} = \frac{i}{E} = f(w)
\]  
(2)

where: \(R\) – unit arc resistance, \(f(W)\) – function of energy \(W\) for unit arc length at time \(dt\).

Taking (1) and (2) into consideration arc discharge conductance \(G\) can be expressed as:

\[
G = f(w) = f(P - P_0) dt
\]  
(3)

or in the differential form:

\[
\frac{1}{i} \frac{dE}{dt} = \frac{1}{E} \frac{dW}{dt} = \frac{df(w)}{dW} (P - P_0)
\]  
(4)

From the above equation depending on taken assumptions the Mayr's model is obtained [5].

Under assumption that the energy in equation (4) is released only by conduction, but other processes of the energy exchange are taken into account in the equivalent conductance \(\gamma\), the following equation is expressed:

\[
\gamma = c_1 e^{\gamma t}
\]  
(5)

where: \(q\) – arc energy, \(q_c\) – energy that changes the arc conductance of \(c\) times, \(c_1\) – constant.

Unit conductance \(G\) and energy \(Q\) of arc discharge can be expressed:

\[
G = 2\pi \int_0^r \gamma r dr
\]  
(6)

\[
Q = 2\pi \int_0^r q r dr
\]  
(7)

Taking the (4) and (7) into account we obtain the Mayr's model of electric arc:

\[
\frac{1}{G} \frac{dG}{dt} = \frac{1}{\tau_M} \left(Ei - \frac{P_0}{P_0 - 1}\right)
\]  
(8)

in which \(\tau_M\) is Mayr’s time constant defined as:

\[
\tau_M = \frac{W_0}{P_0}
\]  
(9)

\(W_0\) is energy content of the arc column and \(P_0\) is the power lost in the static arc by the way of the heat conduction.

The Mayr’s model (8) describes the instantaneous value of electrical conductance of arc, the changes of which are the result of the temperature alteration.

3. The numerical analysis

In the case of gliding discharge, the near-electrode phenomena do not play a significant role and the average unit parameters are regarded as constant along the gliding arc length.

Solutions of differential equations modelling the arc give the dependences on the instantaneous non-linear conductance expressed as follows:

\[
g = g_0 \cdot e^{\gamma t}
\]  
(10)

where: \(u\) – voltage on the arc conductance, \(i\) – instantaneous value of arc current, \(P_0\) – power taken from the discharge, \(g_0\) - initial arc conductance for \(t=0\).

For the numerical analysis of plasma reactor with gliding arc its mathematical model should have following features:

- fulfils basic energetic assumption postulated by Mayr and have suitable dynamic properties,
- describes true course of the static voltage-current characteristic of the arc, especially in the current range not exceeding several amperes,
- takes into consideration changes of the discharge parameters during the cycle of the reactor operation.
permits modelling voltage ignition of the discharge in the cold inter-electrode gap.

In order to fulfil the above-mentioned postulates an attempt to modify the Mayr’s equation was made [1]. Linear dependence between power taken from discharge $p_o$ and its conductance $g$ was formulated:

$$ p_o(g) = a \cdot g + c, \quad (11) $$

where: $a$ and $c$ – constant.

Changes introduced into the Mayr’s model enable suitable modelling of the static characteristics of electric arc discharge with relatively simple mathematical description. Chosen static characteristics of the arc for different $a$ and $c$ constant are presented in Fig. 1.

Since the moment of ignition $t_1$ the value of function $w(t)$ shall be constant until next ignition occurs at the moment $t_2$. Between the moments $t_1$ and $t_2$ conductance course will be determined by thermal phenomena according to the Mayr’s model.

During the operation cycle of the gliding arc discharge its length is subject to considerable changes. It is assumed that the discharge is semicircle-shaped of variable radius.

The results of verifying modelling of the gliding arc discharge powered from sine voltage with internal RL impedance are presented in Fig. 3.

![Fig. 1. Static current characteristics of the simulated electric arc discharge.](image)

Fig. 1. Static current characteristics of the simulated electric arc discharge. Equation (10), together with relation (11), was modified and solved numerically with the aid of a general circuit analysis package PSpice and its analogue behavioural modelling blocks (ABM).

The difficulty concerning electrical ignition of the discharge was solved by introducing additional factor $w(t)$ representing the energy required for discharge reignition.

Finally, modified conductance of the gliding arc discharge $g(t)$ can be expressed as follows:

$$ g = g_0 \cdot e^{-\tau(g) \left( \frac{p(t)}{p_o(g,t)} - 1 \right)} dt \quad (13) $$

If at time $t_1$ conditions for discharge ignition are fulfilled, the function $w(t)$ will reach the value given by (14) and therefore the conductance $g$ of arc discharge will be equal to $g_0$, independently of its value before the moment of ignition.

$$ w(t_1) = -\frac{t_1}{\tau(g) \left( \frac{p(t)}{p_o(g,t)} - 1 \right)} dt \quad (14) $$

The realisation of the gliding arc discharge model by the aid of ABM blocks is presented in Fig. 2.

![Fig. 2. Single-phase RL circuit of the electric arc model.](image)

5. References


